Lesson 024
Method of Moments Estimation

Friday, November 3

Procedures for Generating Estimators

- We know what an estimator (and estimate) is, we know how to compare estimators, we understand bias and variance ... how do we find estimators?
- **Strategy 1:** use what seems reasonable.
  - If we want a mean, use a mean.
- **Strategy 2:** Maximum likelihood estimation.
- **Strategy 3:** Method of Moments Estimators

Method of Moments: Population

- Recall the expectation of a random variable

$$\mu = E[X] = \int_{-\infty}^{\infty} xf(x)dx$$

- This is also called the **first moment.**

- For integer k, we can define the kth moment analogously

$$\mu_k = E[X^k] = \int_{-\infty}^{\infty} x^k f(x)dx$$

Moments: Example

- Suppose that $X \sim N(\mu, \sigma^2)$.

$$E[X] = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = \mu$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = ?$$

Moments: Example, continued

$$E[X^2] = \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx = ?$$

- Recall that $\text{var}(X) = E[X^2] - E[X]^2$.
- Then $E[X^2] = \text{var}(X) + E[X]^2$
- For $X \sim N(\mu, \sigma^2)$ we have $E[X^2] = \sigma^2 + \mu^2$.

If $X \sim \text{Poi}(\lambda)$, what are the first two population moments for X?

$\mu_1 = \lambda$ and $\mu_2 = \lambda$.

0%

$\mu_1 = \lambda$ and $\mu_2 = \lambda + \lambda^2$.

0%

$\mu_1 = \lambda$ and $\mu_2 = \lambda - \lambda^2$.

0%

$\mu_1 = \lambda^2$ and $\mu_2 = \lambda^2$.

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What is the second moment of a binomial distribution with parameters n and p?

$np(1-p)$

0%

np

0%

$(np)^2(1-p)$

0%

$np[(1-p)+np]$

0%

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Method of Moments: Population

- Generally μ_k is going to be a function of the unknown parameter(s), θ.
  - Sometimes, some moments will be constants instead.
  - These are still called moments, but are less useful for us.

Method of Moments: Sample

- Suppose that we have a sample, $\{x_1, \ldots, x_n\}$.
- We have seen the sample mean as analogous to the population mean.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

- What if we generalized this the same way?

Method of Moments: Sample

- For the kth sample moment, we take the sample mean of x^k.

$$\hat{\mu}_k = \frac{1}{n}\sum_{i=1}^{n} x_i^k$$

- With a given sample, numeric values can be assigned to each of these.

Sample Moments: Example

- Suppose that we observe $\{-1, -1, 0, 0, 1, 1\}$.
- We can find the first two sample moments:

$$\widehat{\mu}_1 = \frac{1}{6}(-1 - 1 + 0 + 0 + 1 + 1) = 0$$

$$\widehat{\mu}_2 = \frac{1}{6}\left((-1)^2 + (-1)^2 + 0^2 + 0^2 + 1^2 + 1^2\right) = \frac{2}{3}$$

Suppose you observe $\{-2, 0, 3\}$. What is the second sample moment?

$\frac{1}{3}\left[(-2)^2 + 0^2 + 3^2\right]$

0%

$\frac{1}{3}\left[(-2-\frac{1}{3})^2 + (0-\frac{1}{3})^2 + (3-\frac{1}{3})^2\right]$

0%

$\frac{1}{3}\left[(-2) + 0 + 3\right]$

0%

$\frac{1}{2}\left[(-2-\frac{1}{3})^2 + (0-\frac{1}{3})^2 + (3-\frac{1}{3})^2\right]$

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Method of Moments Estimators

- Suppose we want to estimate $\Theta = (\theta_1, \ldots, \theta_L)$.
- **Step 1:** compute the population moments until you have L available equations.
- **Step 2:** equate the corresponding sample moments, $\widehat{\mu}_j$ with the population moments, $\mu_j(\Theta)$.
- **Step 3:** solve the equations for Θ.

Method of Moments: Example

- What are the method of moments estimators for $\Theta = (\mu, \sigma^2)$ from a $N(\mu, \sigma^2)$ distribution?

- $\mu_1 = \mu$ and $\mu_2 = \sigma^2 + \mu^2$.

- Thus we set $\widehat{\mu}_1 = \mu$ and $\widehat{\mu}_2 = \sigma^2 + \mu^2$.

- Solving gives $\mu = \widehat{\mu}_1$ and $\sigma^2 = \widehat{\mu}_2 - \widehat{\mu}_1^2$.

Method of Moments: Example, Continued

- Solving gives $\mu = \widehat{\mu}_1$ and $\sigma^2 = \widehat{\mu}_2 - \widehat{\mu}_1^2$.

$$\widehat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i = \overline{X}$$

$$\widehat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2 - \overline{X}^2 = \frac{1}{n}\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2$$

Suppose you have a normal population with unknown mean and variance. If your sample mean from $n = 10$ is calculated to be 2 and your sample variance is 4, what is the method of moments estimate for σ^2?

4 — 0%

3.6 — 0%

8 — 0%

2 — 0%

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Suppose that $X \sim \mathrm{Exp}(\lambda)$. A sample of size 50 gives a sample mean of 10. What is the method of moments estimate for λ?

0.1

0%

10

0%

0.01

0%

Not enough information is provided to calculate this.

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Method of Moments Estimators: Key Points

- Generally speaking, method of moments estimators will be biased.
  - For instance, $\widehat{\sigma}^2$.
- If population moments do not exist, method of moments estimators cannot be found.
- Sometimes, the method of moments estimators will produce **impossible values** for the parameters. This is not ideal.